



Contribution vs Attribution.

This article details the methodologies used by Risk101 in calculating both contribution and (performance) attribution, and explains these concepts in layman's terms.

1. Contribution versus attribution. What's the difference?

Contribution forms the foundation of attribution, and allows us to evaluate how shares in a portfolio "contributed" to the overall profit or loss in any reporting period.

CONTRIBUTION does not attempt to compare the performance of the portfolio to any benchmark, nor does it involve any statistical analysis of the performance. This is the role of ATTRIBUTION.

Simplistically speaking, contribution merely looks at the performance of the portfolio from an accounting point of view.

2. Contribution explained.

To explain, let's use the example of the Triple-A hedge fund, who were given \$1 000 000 on January 1 to invest in the stock market.

2.1 On January 1, they invested \$900 000 of their funds in the following three shares, and placed the remaining \$100 000 on overnight call at roughly 6.00%

Share-code	Quantity	Price	Consideration
ABC	100 000	4.00	400 000
DEF	100 000	3.00	300 000
GHI	100 000	2.00	200 000
Cash			100 000
Total:			1 000 000

2.2 On the 31st of January, in line with international hedge-fund reporting conventions, (which require monthly revaluations and reporting) they revalued their portfolio as follows:

Share-code	Quantity	Price	Consideration	Profit
ABC	100 000	4.08	408 000	8 000
DEF	100 000	3.05	305 000	5 000
GHI	100 000	2.06	206 000	6 000
Cash			100 500	500
Revalued total:			1 019 500	19 500

Giving a profit for the month of 19 500 (which includes \$500 interest earned on the cash balance).

2.3 The contribution calculations are the simplest (and most intuitive) of the performance calculations, and they form the foundation for the more complex attribution calculations:

As we can see, we made 19 500 in the month of January, reflecting an increase of 1.95%.

What we want to see though, is the overall return achieved by EACH of the holdings in the portfolio. To the untrained eye, it would appear that share ABC gave us the best return, with a profit of 8 000 for the month of January, but what we really need to do is weight the return by the amount invested in each share.

In each case:

The WEIGHTING of the share is simply its start value divided by the sum of the start values (i.e. the NAV of the portfolio).

The PERCENTAGE RETURN for each share is simply:
 $((\text{End value}/\text{Start value})-1) \times 100$

So the weightings and returns would be calculated as follows:

Share-code	Sum invested on 1 January	Valuation as at 31 January	Weighting	Percentage return
ABC	400 000	408 000	40.00%	2.00%
DEF	300 000	305 000	30.00%	1.67%
GHI	200 000	206 000	20.00%	3.00%
Cash	100 000	100 500	10.00%	0.50%

From the above example, we can see very easily that while share ABC gave us the lion's share of the profits in January, in fact it was share GHI that was the best performer for the month. Using these sector weights and returns, it is now possible to calculate the CONTRIBUTION of each share to the total return. The contribution is simply the weight multiplied by the return:

2.3.1	Share-code	Weighting	Percentage Return	Contribution
	ABC	40.00%	2.00%	0.80%
	DEF	30.00%	1.67%	0.50%
	GHI	20.00%	3.00%	0.60%

	Subtotal for the securities, excluding cash :			1.90%
	Cash	10.00%	0.50%	0.05%

	Total :			1.95%

-Which ties back to our original number further up. (This figure of 1.95% is also referred to as 195 basis points.)

2.4 Weighted Sums

The contribution calculations now lead us on to WEIGHTED SUMS, which are used extensively in the analysis of investment performance.

What is a weighted sum? We saw an example in the preceding paragraph where the total contribution of all the shares in the portfolio was

$$(40\% \times 2\%) + (30\% \times 1.67\%) + (20\% \times 3\%) + (10\% \times 0.5\%) = 1.95\%.$$

The contribution calculation is done by multiplying each sector's weight by its return, and then summing the results.

BUT: If only it was that easy.

The big problem with calculating returns using monthly data is that we only have a "snapshot" of the portfolio from the end of each month, as in the case of Triple-A investments above.

What if Triple-A actually purchased a shareholding in a different security, namely LMN, but sold it before month end? In this case we would not be able to weight LMN's return using monthly data, as the holding was square (zero) at the end of January.

As a result of these types of problems it is widely regarded that the most accurate way to calculate the returns of a portfolio is to use DAILY data, but to reset the figures at the end of each month. (When profit figures are reported, and any capital inflows and outflows occur.)

The Risk101 attribution module uses daily data, and to obtain daily holdings for each portfolio under review, uses a methodology called "portfolio reconstruction".

What is portfolio reconstruction?

In effect Risk101 grabs a snapshot of the portfolio at the beginning of the month, and then finds all trades done for the portfolio for the entire month.

It then sorts the trades by dealing date, and allocates each trade to the correct day on which it was dealt.

Using the snapshot of the portfolio at the beginning of the month, and the trades for day-1, it is then able to recreate the portfolio holdings as at the end of trade on day-1. These holdings are then rolled forward to day-2, the trades for day-2 are applied, and so on for each day of the month.

Once Risk101 has recreated a snapshot of the portfolio for each day of the month, we are then able to recalculate the returns, but on a DAILY WEIGHTED AVERAGE basis.

Let's use the example from Triple-A investments above again:

2.5 As before, on January 1, they invested their funds in the following three shares

2.5.1	Share-code	Quantity	Price	Consideration	Date Purchased
	ABC	100 000	4.00	400 000	1 January
	DEF	100 000	3.00	300 000	1 January
	GHI	100 000	2.00	200 000	1 January
	Cash			100 000	1 January
	Total :			1 000 000	

.... and put the remaining 100 000 on overnight call at 6.00%

Then on 8 January the fund manager purchased 10 000 shares in LMN at 10.00 per share, so at the close of trade on 8 January their portfolio holdings were as follows:

2.5.2	Share-code	Quantity	Price	Consideration	Date Purchased
	ABC	100 000	4.00	400 000	1 January
	DEF	100 000	3.00	300 000	1 January
	GHI	100 000	2.00	200 000	1 January
	LMN	10 000	10.00	100 000	8 January
	Cash			-	
	Total :			1 000 000	

The fund manager at Triple-A did nothing for a week, and then on 15 January sold his shareholding in LMN for 10.20 which yielded a profit of 2000.

Now, when we redo the calculations in 2.3 above, we can see that the assumptions that we made are far too simplistic.

To recap:

2.5.3	Share code	Sum invested on 1 January	Valuation as at 31 January	Weighting	Percentage return
	ABC	400 000	408 000	40.00%	2.00%
	DEF	300 000	305 000	30.00%	1.67%
	GHI	200 000	206 000	20.00%	3.00%
	LMN	?	?	?	?
	Cash	?	?	?	?

As there are "holes" in the data for the LMN security, as well as interest earned on the cash, what we do now is look at the contributions of each share from TWO DIFFERENT POINTS OF VIEW:

VIEW 1: We calculate the percentage returns on a profit/NAV basis, (not a weighted-average basis) which will give us the IDENTICAL overall answers as 2.31

2.5.4	Share-code	Profit for January	Portfolio NAV as at 31 January	Percentage return
	ABC	(8 000 /	1 000 000) x 100	0.8000%
	DEF	(5 000 /	1 000 000) x 100	0.5000%
	GHI	(6 000 /	1 000 000) x 100	0.6000%
		-----		-----
		19 000	Subtotal (Same as 2.31) :	1.9000%
	Plus :			
	LMN	(2 000 /	1 000 000) x 100	0.2000%
	Cash	(395 /	1 000 000) x 100	0.0395% (24 days int @ 6%)
		-----		-----
	TOTALS	(21 395 /	1 000 000) x 100	2.1395%

BUT : As we can see, while these answers give us an accurate reflection of the CONTRIBUTION of each share in the portfolio, they do not give us any idea of the PERFORMANCE of each individual share in the portfolio.

So what we now do is:

VIEW 2: We calculate the average returns, but weight them by the number of days that we were actually invested in the security for:

(Remember that the percentage-returns that we have calculated so far are MONTHLY returns, ie : They have not been annualised. To be really precise, the returns that we have calculated are 31-day returns)

Referring back to 2.5.3, the monthly return is calculated as follows:

MPL = Profit/loss for the month

NDP = No of days in period

AMI = Amount invested in security

MR = Monthly return = $[(AMI+MPL)/AMI]-1 \times 100$

WAR = Weighted average return = $(MR \times NDP) / \text{No. of days that we were invested for.}$

And we apply the formula to Triple-A's portfolio:

2.5.5	Share-code	Monthly return (MR)	x	Number of days in January (NDP)	/	No. of days that we were invested for	=	Weighted average return (WAR)
	ABC	(2.00%	x	31)	/	31	=	2.00%
	DEF	(1.67%	x	31)	/	31	=	1.67%
	GHI	(3.00%	x	31)	/	31	=	3.00%
	LMN	(2.00%	x	31)	/	7	=	8.85%
	Cash	(0.0395%	x	31)	/	24	=	0.05%

From this, we can see that our investment in LMN was by far the best performer for the month of January, because our opportunity cost was only 7 days, compared to 31 days for the other securities in the portfolio.

(Another interesting point to note is that the cash return was again 0.05% which is correct, given that it was earning a consistent rate of interest for the month.)

UNFORTUNATELY:

The formula we have used in 2.5.5 above is still not sophisticated enough, because in modern portfolio trading, the manager will often be trading in and out of shares on a daily basis, and you will regularly find that the investment in any one security will change on a day-to-day basis.

2.5.6 So let's take it a step further, and break down the formula to make it easier to understand....

(MPL = Profit/loss for the month)

(NDP = No of days in period)

AI = Average investment
= (Investment on day 1 + Investment on day 2 + etc. + Investment on last day)/NDP
WAR = Weighted average return = [(AI+MPL)/AI]-1] x 100

Now let's apply it to our Triple-A example again. (It is summarised - use a calculator to work out each line in detail)

2.5.7	Share-code	Profit for January (MPL)	Average investment for January (AI)	Weighted average return (WAR)
	ABC	8 000	(400 000 x 31)/31 = 400 000	2.00%
	DEF	5 000	(300 000 x 31)/31 = 300 000	1.67%
	GHI	6 000	(200 000 x 31)/31 = 200 000	3.00%
	LMN	2 000	(100 000 x 7)/31 = 22 581	8.85%
	Cash	395	(100 000 x 24)/31 = 77 419	0.50%

As you can see, the formula to calculate the weighted average return using daily investment figures gives EXACTLY the same answers as the formula in 2.5.5 but allows for the daily movements in and out of securities, and is consequently far more accurate.

Now we can move onto the "Return on investment ratio", or the ROI, which is NEARLY IDENTICAL to the Weighted average return, except that the answer is expressed as a ratio, rather than a percentage.

Again, the ROI gives us an exact idea of how each security has performed, given the amount of both money AND time that we have been invested in the security.

The formula is as follows: $ROI = (AI+MPL)/AI$

The ROI's for Triple-A's portfolio for the month of January would be as follows:

Share-code	Return on investment ratio
ABC	1.0200
DEF	1.0167
GHI	1.0300
LMN	1.0885
Cash	1.0050

Some important notes:

- Risk101 always includes costs, such as exchange-fees, relevant state taxes etc., as these numbers can be quite substantial, and can make a significant impact on the ratios.
- When calculating the performance of SHORT positions, we use ABSOLUTE daily investment figures, allowing us to compare the performance of short positions against long positions.

3. The Return-on-exposure ratio.

The ROE ratio is calculated on an identical basis to the ROI ratio, using daily weighted numbers, except we use the the daily REVALUED EXPOSURE of each holding, rather than the amount invested in the holding.

When looking at vanilla underlying instruments such as normal equities, these numbers will be quite similar to the ROI numbers, but when the portfolio contains options or hybrids which do not have a delta of 1.00, the ROE can swing around wildly, and provides an accurate measure of the returns, given the risk being taken.

With respect to derivatives: The delta-adjusted cash exposure is first calculated, before "collapsing" or "merging" the derivative into the underlying security.

The ROE ratio also allows the investor to quickly see whether or not the fund manager is using derivatives for hedging or gearing purposes.

And with these numbers available, we start to now enter into the realms of attribution.

4. Attribution explained.

4.1 Benchmarks and notional portfolios

We now know how to calculate the ROI, the ROE, and the WAR, but how has our manager's performance at Triple-A compared to other investors in the market?

As we do not have access to all the other investors' portfolios, the easiest way is to compare the performance of our fund against a benchmark.

Also referred to as "notional portfolios", this concept is critical to understanding performance attribution.

A benchmark consists of combining a set of shares together, (normally weighted by the number of shares in issue) to create a portfolio of the top 100 or 200 shares on an exchange.

On the issue of weighting: Nearly all exchanges also multiply the number of shares in issue by what is known as the "free-float" factor when calculating a benchmark. This factor represents the no. of shares that are freely available to trade in the open marketplace.

An example of this would be : If a listed company is controlled by a family that has a 15% shareholding in the business, these shares would very rarely be traded, and the exchange may decide to exclude them. Consequently the share would have a free-float factor of 0.85, or 85.00%

The "free-float" factor is also known as the "DC" factor, and may be expressed as a number between 0.00 and 1.00, or as a percentage between 0.00% and 100.00%

But back to benchmarks

Using the benchmark, we can see if our fund manager has out- or underperformed the benchmark in any reporting period. Again, all the problems referred to in 2.5 above rear their ugly head again. It is absolutely pointless trying to calculate attribution using monthly figures, for all the same reasons that we referred to in 2.5 above.

As a result of this, Risk101 calculates attribution on a DAILY basis, and calculates the relative performance of each security in the portfolio against the following three standards:

4.1.1 The overall benchmark (or "notional portfolio").

4.1.2 The sector that the security forms part of, within the benchmark. (eg : Energy, mining, financials, etc.)

Please note: If the share does not form part of the benchmark index, then this number will obviously not be calculated.

4.1.3 The security's performance against itself.

It might seem a bit odd at first, and as you would assume, if you purchased a security and merely held onto it, your shareholding could neither out- or underperform itself, BUT If the fund manager is trading in and out of the same security on a daily basis, or is using options or hybrids for hedging or gearing purposes, then these figures can be substantially different, and allow us to see whether or not the trading decisions that the fund manager made were effective.

4.2 Asset allocation and weighting

Another important number supplied by the attribution module is the weighting of each security in the portfolio compared to the weighting of that security in the benchmark index.

Let's use Triple-A's portfolio again as our example. To keep it simple, we will look at the whole month of January, and assume that our fund manager held the shares for the entire month:

(Please note that the "index weighting" number is purely an example, and cannot be implied from any of the portfolio information. We have merely assumed that three of the four shares in our portfolio form an index)

* Convention states that you always take the weighting at the start of the period.

Share code	Quantity	Consid -eratr.	Price @start (PS)	Price @end (PE)	%-age return (RET)	*Index weight (IW)	*Fund weight (FW)	Over/ under (OVUN)	Relative return (RRET)	Attrib- ution (ATTR)
ABC	100 000	400 000	4.00	4.07	1.75%	50.00%	40.00%	-10.00%	0.7925%	-0.07925%
DEF	100 000	300 000	3.00	3.04	1.33%	25.00%	30.00%	5.00%	0.3725%	0.01863%
GHI	100 000	200 000	2.00	1.98	-1.00%	25.00%	20.00%	-5.00%	-1.9575%	0.09788%
LMN	10 000	100 000	10.00	10.22	2.20%	-	10.00%	10.00%	1.2425%	0.12425%
	-----	1 000 000				100.00%	100.00%			0.16151%

Weighted index return (WIR) : 0.9575%

Weighted fund return (WFR) : 1.1190%

Outperformance (OPR) : 0.1615%

So the formulae would be as follows:

1. RET = ((PE-PS)/PS) x 100
2. WIR = Sum of [RET x IW] for each security forming part of the index.
3. WFR = Sum of [RET x FW] for each security in the portfolio.
4. OPR = WFR - WIR
5. OVUN = FW - IW
6. RRET = RET - WIR
7. ATTR = RRET x OVUN

A couple of points to note:

- 4.2.1 You will see that the security LMN does NOT form part of the WIR calculation but DOES form part of WFR. A purist might say that you cannot compare WIR to WFR, as they have different constituents, but it was the fund managers CHOICE to invest in a non-benchmark stock, and consequently the performance numbers are assumed to be comparable.
- 4.2.2 Before calculating the attribution numbers, Risk101 always calculates the delta-adjusted-exposure of any derivatives/options first. It then uses this number to collapse/merge the derivative into its underlying security. Attribution is then calculated using the merged holdings.
- 4.2.3 Please remember that ALL the numbers above are calculated against both the benchmark, as well as all the sectors that form the benchmark. Risk101 allows you to drill-down to the 2nd tier when running attribution.
- 4.2.4 From this we can see that the actual profit that we made from each share for the month of January is of no interest to us. (These numbers have already been analysed by the contribution statistics) What we can now see is how each of the shares in our portfolio performed relative to the benchmark.
- 4.2.5 Many users of the Risk101 product do not book trades into the system, but rather import their portfolio balances on a daily basis for risk analysis purposes. In the latter case, the attribution module uses the daily balances, (rather than reconstructing the balances from trades) but STILL ASSUMES monthly reporting, and will reset profit and losses on the last calendar day of each month under review.
- 4.2.6 Throughout the attribution module, Risk101 assumes a seven-day week, and forces in Fridays' holdings for both Saturday and Sunday.

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